MATH 464 (THEORY OF PROBABILITY) HOMEWORK 10

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(1) Let $X_1, \dots, X_n$ be independent identically distributed (i.i.d.) random variables random variables from $U(0,1)$. Let $U := \min\{X_1, \dots, X_n\}$, and let $Y_n = nU$. Prove that
$$\lim_{n\to\infty} \mathbb{P}(Y_n > x) = e^{-x}.$$
Here we say that Y_n is asymptotically Exp(1).

(2) Random variables X and Y have joint density function
$$f_{X,Y}(x,y) = \begin{cases} c(x^2 + \frac{1}{2}xy) & 0 < x < 1,\ 0 < y < 2, \\ 0 & \text{otherwise.} \end{cases}$$
(a) Find the value of the constant c.
(b) Find $\mathbb{P}(Y \le 2X)$.

(3) The following is a joint probability density function, find the value of the constant c.
$$f(x,y) = \frac{c}{(1 + x^2 + y^2)^{3/2}} \quad \text{for } x, y \in \mathbb{R}^2.$$

(4) Let X and Y have joint density function
$$f(x,y) = \begin{cases} \frac{1}{4}(x + 4y)e^{-(x+y)} & \text{if } x, y \ge 0 \\ 0 & \text{otherwise.} \end{cases}$$
Find the marginal density function of Y.

(5) Suppose that X and Y are uniformly distributed on $A = \{(x,y) \in \mathbb{R}^2;\ 0 < y < x < 1\}$, find $\mathbb{P}(X - Y > z)$, where z is a constant.

(6) Suppose X and Y are independent standard normal random variables. Find the distribution of $W = (X^2 + Y^2)^{1/2}$.
Remark: This is called the *Rayleigh distribution*.

(7) Let $X_1, \dots, X_n$ are i.i.d. random variables, each with probability distribution F and probability density function f. Define
$$U = \max\{X_1, \dots, X_n\}, \quad V = \min\{X_1, \dots, X_n\}.$$
(a) Find the distribution function and the density function of U and of V.
(b) Show that the joint density function of U and V is
$$f_{U,V}(u,v) = n(n-1)f(u)f(v)\big[F(v) - F(u)\big]^{n-1}, \quad \text{if } u < v.$$